

TE_xMaT | Texas Examinations for Master Teachers

Preparation Manual



o87 Master Mathematics Teacher EC-4

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PREFACE

The Texas Examinations for Master Teachers (TExMaT) Program has its origins in legislation passed in 1999 (House Bill 2307) that required the creation of the Master Reading Teacher (MRT) Certificate, the development of standards for the certificate, and the development of a Master Reading Teacher examination. In 2001, the Texas legislature passed legislation creating two additional categories of Master Teacher Certificates, the Master Mathematics Teacher (three certificates: Early Childhood–Grade 4, Grades 4–8, and Grades 8–12) and Master Technology Teacher (Early Childhood–Grade 12).

The Master Mathematics Teacher Certificate was created by the 77th Texas Legislature "to ensure that there are teachers with special training to work with other teachers and with students in order to improve student mathematics performance. . . ." A Master Mathematics Teacher will be an individual who holds a Master Mathematics Teacher Certificate and whose primary duties are to teach mathematics and to serve as a mathematics teacher mentor to other teachers.

A Master Mathematics Teacher Certificate may be obtained by individuals who:

- hold a teaching certificate,
- have at least three years of teaching experience,
- complete an SBEC-approved Master Mathematics Teacher preparation program, AND
- pass the TExMaT Master Mathematics Teacher EC–4, 4–8, or 8–12 certification examination.

The development of the educator standards for the Master Mathematics Teacher Certificates was completed in November 2001. The first SBEC-approved Master Mathematics Teacher preparation programs became available during the summer of 2002. The TExMaT Master Mathematics Teacher certification examinations will be administered for the first time in the summer of 2003.

This manual is designed to help examinees prepare for the new Master Mathematics Teacher EC–4 test. Its purpose is to familiarize examinees with the competencies to be tested, test item formats, and pertinent study resources. Educator preparation program staff may also find this information useful as they help examinees prepare for careers as Texas Master Teachers.

More information about the new TExMaT tests and the educator standards can be found at <http://www.sbectx.us>.

KEY FEATURES OF THE MANUAL

List of competencies that will be tested

Strategies for answering test questions

Sample test items and answer key

If you have questions after reading this preparation manual, please contact the State Board for Educator Certification, Office of Accountability at 1-512-238-3200.

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SECTION I

THE NEW TExMaT TESTS

Successful performance on the TExMaT examination is required for the issuance of a Texas Master Teacher certificate. Each TExMaT test is a criterion-referenced examination designed to measure the knowledge and skills delineated in the corresponding TExMaT test framework. Each test framework is based on standards that were developed by Texas educators and other education stakeholders.

Each TExMaT test is designed to measure the requisite knowledge and skills that an initially certified Texas Master Teacher in this field must possess. This test includes multiple-choice items (questions) as well as a case study assignment for which candidates will construct a written response.

Development of the New TExMaT Tests

Committees of Texas educators and interested citizens guide the development of the new TExMaT tests by participating in each stage of the test development process. These working committees are comprised of Texas educators from public and charter schools, faculty from educator preparation programs, education service center staff, representatives from professional educator organizations, content experts, and members of the business community. The committees are balanced in terms of position, affiliation, years of experience, ethnicity, gender, and geographical location. The committee membership is rotated during the development process so that numerous Texas stakeholders may be actively involved. The steps in the process to develop the TExMaT tests are described below.

1. **Develop Standards.** Committees are convened to recommend what an initially certified Master Teacher in this field should know and be able to do. To ensure vertical alignment of standards across the range of instructional levels, individuals with expertise in early childhood, elementary, middle, or high school education meet jointly to articulate the critical knowledge and skills for a particular content area. Participants begin their dialogue using a "clean slate" approach with the Texas Essential Knowledge and Skills (TEKS) as the focal point. Draft standards are written to incorporate the TEKS and to expand upon that content to ensure that an initially certified Master Teacher in this field possesses the appropriate level of both knowledge and skills to instruct successfully.
2. **Review Standards.** Committees review and revise the draft standards. The revised draft standards are then placed on the SBEC Web site for public review and comment. These comments are used to prepare a final draft of the standards that will be presented to the SBEC Board for discussion, the State Board of Education (SBOE) for review and comment, and the SBEC Board for approval.
3. **Develop Test Frameworks.** Committees review and revise draft test frameworks that are based on the standards. These frameworks outline the specific competencies to be measured on the new TExMaT tests. The TExMaT competencies represent the critical components of the standards that can be measured with either a paper-and-pencil-based or a computer-based examination, as appropriate. Draft frameworks are not finalized until after the standards are approved and the job analysis/content validation survey (see #4) is complete.

4. **Conduct Job Analysis/Content Validation Surveys.** A representative sample of Texas educators who practice in or prepare individuals for each of the fields for which a Master Teacher certificate has been proposed are surveyed to determine the relative job importance of each competency outlined in the test framework for that content area. Frameworks are revised as needed following an analysis of the survey responses.
5. **Develop and Review New Test Items.** The test contractor develops draft items (multiple-choice and case study assignments) that are designed to measure the competencies described in the test framework. Committees review the newly developed test items that have been written to reflect the competencies in the new test frameworks and may accept, revise, or reject test items. Committee members scrutinize the draft items for appropriateness of content and difficulty; clarity; match to the competencies; and potential ethnic, gender, and regional bias.
6. **Conduct Pilot Test of New Test Items.** All of the newly developed test items that have been deemed acceptable by the item review committees are then administered to an appropriate sample of candidates for certification.
7. **Review Pilot Test Data.** Pilot test results are reviewed to ensure that the test items are valid, reliable, and free from bias.
8. **Administer New TExMaT Tests.** New TExMaT tests are constructed to reflect the competencies, and the tests are administered to candidates for certification.
9. **Set Passing Standard.** A Standard Setting Committee convenes to review performance data from the initial administration of each new TExMaT test and to recommend a final passing standard for that test. SBEC considers this recommendation as it establishes a passing score on the test.

Taking the TExMaT Master Mathematics Teacher Test and Receiving Scores

Please refer to the current TExMaT registration bulletin for information on test dates, sites, fees, registration procedures, and policies.

You will be mailed a score report approximately four weeks after each test you take. The report will indicate whether you have passed the test and will include:

- a total test *scaled* score. Scaled scores are reported to allow for the comparison of scores on the same content-area test taken on different test administration dates. The total scaled score is not the percentage of items answered correctly and is not determined by averaging the number of questions answered correctly in each domain.
 - For all TExMaT tests, the score scale is 100–300 with a scaled score of 240 as the minimum passing score. This score represents the minimum level of competency required to be a Master Teacher in this field in Texas public schools.
- a holistic score for your response to the case study assignment.
- your performance in the major content domains of the test and in the specific content competencies of the test.
 - This information may be useful in identifying strengths and weaknesses in your content preparation and can be used for further study or for preparing to retake the test.
- information to help you understand the score scale and interpret your results.

You will not receive a score report if you are absent or choose to cancel your score.

Additionally, unofficial score report information will be posted on the Internet on the score report mailing date of each test administration. Information about receiving unofficial scores via the Internet and other score report topics may be found on the SBEC Web site at www.sbec.state.tx.us.

Educator Standards

Complete, approved educator standards are posted on the SBEC Web site at www.sbec.state.tx.us.

SECTION II

USING THE TEST FRAMEWORK

The Texas Examinations for Master Teachers (TExMaT) test measures the content and professional knowledge required of an initially certified Master Teacher in this field. This manual is designed to guide your preparation by helping you become familiar with the material to be covered on the test.

When preparing for this test, you should focus on the competencies and descriptive statements, which delineate the content that is eligible for testing. A portion of the content is represented in the sample items that are included in this manual. These test questions represent only a *sample* of items. Thus, your test preparation should focus on the complete content eligible for testing, as specified in the competencies and descriptive statements.

Organization of the TExMaT Test Framework

The test framework is based on the educator standards for this field.

The content covered by this test is organized into broad areas of content called domains. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of competencies. Each competency is composed of two major parts:

1. the *competency statement*, which broadly defines what an initially certified Master Teacher in this field should know and be able to do, and
2. the *descriptive statements*, which describe in greater detail the knowledge and skills eligible for testing.

The educator standards being assessed within each domain are listed for reference at the beginning of the test framework. These are then followed by a complete set of the framework's competencies and descriptive statements.

An example of a competency and its accompanying descriptive statements is provided on the next page.

Sample Competency and Descriptive Statements

Master Mathematics Teacher EC–4

Competency:

The Master Mathematics Teacher EC–4 understands the structure of number systems, the development of a sense of quantity, and the relationship between quantity and symbolic representations.

Descriptive Statements:

The Master Mathematics Teacher:

- Compares and contrasts numeration systems.
- Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.
- Demonstrates a sense of quantity and estimation for the real numbers (i.e., whole numbers, integers, rational and irrational numbers).
- Demonstrates an understanding of a variety of models for representing real numbers (e.g., fraction strips, diagrams, base ten blocks, number lines, sets).
- Demonstrates an understanding of different representations of equivalent rational numbers.
- Selects appropriate representations of real numbers (e.g., expanded notation, fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Demonstrates, explains, and uses models to show how some situations that have no solution in the whole, integer, or rational number systems have solutions in the real number system.

Studying for the TExMaT Test

The following steps may be helpful in preparing for the TExMaT test.

1. Identify the information the test will cover by reading through the test competencies (see the following pages in this section). *Within each domain* of this TExMaT test, each competency will receive approximately equal coverage.
2. Read each competency with its descriptive statements in order to get a more specific idea of the knowledge you will be required to demonstrate on the test. You may wish to use this review of the competencies to set priorities for your study time.
3. Review the "Preparation Resources" section of this manual for possible resources to consult. Also, compile key materials from your preparation coursework that are aligned with the competencies.
4. Study this manual for approaches to taking the test.
5. When using resources, concentrate on the key ideas and important concepts that are discussed in the competencies and descriptive statements.

NOTE: This preparation manual is the only TExMaT test study material endorsed by SBEC for this field. Other preparation materials may not accurately reflect the content of the test or the policies and procedures of the TExMaT Program.

TEST FRAMEWORK FOR MASTER MATHEMATICS TEACHER EC–4

Domain I **Number Concepts: Content, Instruction, and Assessment** (approximately 23% of the test)

Standards Assessed:

Standard I: Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Domain II **Patterns and Algebra: Content, Instruction, and Assessment** (approximately 18% of the test)

Standards Assessed:

Standard II: Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

**Domain III Geometry and Measurement: Content, Instruction, and Assessment
(approximately 23% of the test)**

Standards Assessed:

Standard III: Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

**Domain IV Probability and Statistics: Content, Instruction, and Assessment
(approximately 18% of the test)**

Standards Assessed:

Standard IV: Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

**Domain V Mathematical Processes, Perspectives, Mentoring, and Leadership
(approximately 18% of the test)**

Standards Assessed:

Standard V: Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.

Standard IX: Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.

Standard X: Mathematical Perspectives: The Master Mathematics Teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics, and the evolving nature of mathematics and mathematical knowledge.

DOMAIN I—NUMBER CONCEPTS: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 001

The Master Mathematics Teacher EC–4 understands the structure of number systems, the development of a sense of quantity, and the relationship between quantity and symbolic representations.

The Master Mathematics Teacher:

- Compares and contrasts numeration systems.
- Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.
- Demonstrates a sense of quantity and estimation for the real numbers (i.e., whole numbers, integers, rational and irrational numbers).
- Demonstrates an understanding of a variety of models for representing real numbers (e.g., fraction strips, diagrams, base ten blocks, number lines, sets).
- Demonstrates an understanding of different representations of equivalent rational numbers.
- Selects appropriate representations of real numbers (e.g., expanded notation, fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Demonstrates, explains, and uses models to show how some situations that have no solution in the whole, integer, or rational number systems have solutions in the real number system.

Competency 002

The Master Mathematics Teacher EC–4 understands number operations and computational algorithms.

The Master Mathematics Teacher:

- Recognizes, models, and describes different ways to interpret the four basic operations involving real numbers (e.g., whole numbers, integers, fractions, decimals).
- Analyzes, describes, and connects relationships among number properties, operations, and algorithms involving the four basic operations with real numbers (e.g., whole numbers, integers, fractions, decimals).
- Recognizes error patterns that often occur when students use algorithms to perform operations.
- Recognizes and analyzes appropriate alternative algorithms for the four basic operations with whole numbers, fractions, and decimals.
- Translates among concrete, pictorial, and symbolic representations of the four basic operations with real numbers, and recognizes these concepts and relationships in real-life situations.

Competency 003

The Master Mathematics Teacher EC–4 understands the basic ideas of number theory and uses numbers to model and solve problems within and outside of mathematics.

The Master Mathematics Teacher:

- Describes, sorts, and classifies numbers as prime or composite, and applies the concepts of prime and composite numbers in problem situations.
- Recognizes greatest common denominators and least common multiples and uses these concepts to solve problems.
- Applies the concept of prime factorization to solve problems.
- Applies knowledge of place value and other number properties to develop techniques of mental mathematics and computational estimation.
- Recognizes problem situations in which use of particular mathematical operations (e.g., multiplication, division) would be useful or necessary.
- Uses integers, fractions, decimals, and percents to solve problems in a variety of real-world situations.

Competency 004

The Master Mathematics Teacher EC–4 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop number concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use number concepts.
- Recognizes and uses the vertical alignment of number concepts across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of number concepts.
- Uses evidence of students' current understanding of number concepts to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within number concepts.
- Designs challenging and engaging problem-solving tasks that develop number-concepts content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates number concepts within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of number concepts.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 005

The Master Mathematics Teacher EC–4 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of number concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve number concepts, skills, and procedures.
- Knows how to teach number concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of number concepts.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning number concepts.
- Evaluates and modifies instruction to improve learning of number concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN II—PATTERNS AND ALGEBRA: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 006

The Master Mathematics Teacher EC–4 understands and uses mathematical reasoning to identify, extend, and analyze patterns, and understands the relationships among variables, expressions, equations, relations, and functions.

The Master Mathematics Teacher:

- Uses inductive reasoning to identify and generalize patterns with numbers, concrete models, geometric figures, tables, graphs, and algebraic expressions.
- Uses a variety of number patterns (e.g., fact families, number charts, multiplication by powers of 10) to explore number properties.
- Formulates rules to describe and construct sequences using concrete models, geometric figures, tables, graphs, and algebraic expressions.
- Demonstrates an understanding of making, testing, validating, and using conjectures about patterns and relationships in data presented in tables, sequences, or graphs.
- Applies relations and functions to represent mathematical and real-world situations.
- Translates problem-solving situations into expressions and equations.

Competency 007

The Master Mathematics Teacher EC–4 understands and uses linear functions to model and solve problems using a variety of methods, including algebra.

The Master Mathematics Teacher:

- Demonstrates an understanding of the concept of linear function using concrete models, tables, graphs, and symbolic and verbal representations.
- Analyzes the relationship between a linear function or relation and its graph.
- Uses linear functions and relations to model problems.
- Uses tables, graphs, and algebraic techniques to solve linear equations.
- Gives appropriate justification for the manipulation of algebraic expressions and equations in one variable.
- Models and solves problems, including proportion problems, using concrete, geometric, tabular, graphic, and algebraic methods.
- Distinguishes between linear and nonlinear functions.

Competency 008

The Master Mathematics Teacher EC–4 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop patterns and algebra concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use patterns and algebra.
- Recognizes and uses the vertical alignment of patterns and algebra across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that students develop accuracy and fluency of patterns and algebra.
- Uses evidence of students' current understanding of patterns and algebra to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within patterns and algebra.
- Designs challenging and engaging problem-solving tasks that develop patterns and algebra content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates patterns and algebra concepts within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of patterns and algebra.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 009

The Master Mathematics Teacher EC–4 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of patterns and algebra concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve patterns and algebra concepts, skills, and procedures.
- Knows how to teach patterns and algebra concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of patterns and algebra concepts.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning patterns and algebra concepts.
- Evaluates and modifies instruction to improve learning of patterns and algebra concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN III—GEOMETRY AND MEASUREMENT: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 010

The Master Mathematics Teacher EC–4 understands measurement as a process.

The Master Mathematics Teacher:

- Identifies attributes to be measured, quantifies the attributes by selecting and using appropriate units (standard and nonstandard), and communicates information about the attributes using the unit measure.
- Explains and illustrates the use of numbers and units of measurement for quantities such as length, perimeter, area, volume, temperature, percent, speed, and acceleration.
- Uses conversions within and between different measurement systems.
- Applies measurement concepts and dimensional analysis to derive units and formulas for a variety of situations, including average rates of change of one variable with respect to another.
- Uses methods of approximation and estimates the effects of error on measurement.
- Applies measurement to solve problems in real-world situations.

Competency 011

The Master Mathematics Teacher EC–4 understands the basic concepts and applications of Euclidean geometry.

The Master Mathematics Teacher:

- Demonstrates an understanding of properties of and relationships among points, lines, planes, angles, polygons, and circles.
- Demonstrates an understanding of geometric constructions using a compass and straightedge, reflection devices, and other appropriate technologies.
- Uses logical reasoning to analyze geometric relationships.
- Finds length, perimeter, area, and volume of geometric objects.
- Analyzes and solves problems involving one-, two-, and three-dimensional objects (e.g., lines, angles, circles, polygons, cylinders, cones, pyramids, prisms, spheres).
- Analyzes the relationship among three-dimensional objects and related two-dimensional representations (e.g., perspectives, projections, cross sections, nets) and uses these representations to solve problems.
- Demonstrates an understanding of the use of manipulatives and appropriate technology to investigate and illustrate geometric relationships.
- Applies geometry to model and solve problems in real-world situations.

Competency 012

The Master Mathematics Teacher EC–4 understands transformational and coordinate geometry and connects geometry with other topics in the mathematics curriculum.

The Master Mathematics Teacher:

- Uses visualization skills, spatial reasoning, and geometric modeling to investigate and describe shape in terms of dimension, direction, orientation, and perspective.
- Uses translations, rotations, and reflections to illustrate congruencies and properties of congruent figures.
- Demonstrates an understanding of the properties of similar figures.
- Uses transformations to explore symmetries of figures.
- Analyzes tessellations and shows how they can be used to illustrate geometric concepts, properties, and relationships.
- Specifies locations and describes spatial relationships using coordinate geometry.
- Relates geometry to algebra by using the Cartesian coordinate system and uses this relationship to solve problems.
- Makes connections among geometric ideas and number concepts, measurement, probability and statistics, algebra, and analysis.

Competency 013

The Master Mathematics Teacher EC–4 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop geometry and measurement concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use geometry and measurement.
- Recognizes and uses the vertical alignment of geometry and measurement across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of geometry and measurement.
- Uses evidence of students' current understanding of geometry and measurement to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within geometry and measurement.
- Designs challenging and engaging problem-solving tasks that develop geometry and measurement content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates geometry and measurement within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of geometry and measurement.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 014

The Master Mathematics Teacher EC–4 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of geometry and measurement concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve geometry and measurement concepts, skills, and procedures.
- Knows how to teach geometry and measurement concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of geometry and measurement.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning geometry and measurement.
- Evaluates and modifies instruction to improve learning of geometry and measurement concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN IV—PROBABILITY AND STATISTICS: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 015

The Master Mathematics Teacher EC–4 understands the theory of probability.

The Master Mathematics Teacher:

- Uses data collection, experiments, and simulations to explain concepts of probability.
- Uses probability to describe the expected outcomes of simple and compound events.
- Determines probabilities by constructing a sample space that models a situation.
- Solves problems involving counting combinations and permutations.
- Generates and uses probability models to represent or simulate a situation.
- Uses probability language to make observations and draw conclusions.

Competency 016

The Master Mathematics Teacher EC–4 understands how to collect, describe, display, and draw appropriate inferences from data.

The Master Mathematics Teacher:

- Uses appropriate tabular and graphical displays (e.g., frequency tables, histograms, pie charts) for categorical and numerical data.
- Calculates mean, median, mode, and range to describe a set of data.
- Interprets measures of central tendency and dispersion (i.e., range, percentile, interquartile range, standard deviation) to describe a set of data.
- Describes and analyzes relationships between sets of data using techniques such as scatterplots and trend lines.
- Applies knowledge of designing, conducting, analyzing, and interpreting statistical experiments to investigate real-world problems.
- Recognizes the misuse of statistics.
- Uses appropriate language to communicate the results of a statistical investigation.

Competency 017

The Master Mathematics Teacher EC–4 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop probability and statistics concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use probability and statistics.
- Recognizes and uses the vertical alignment of probability and statistics across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of probability and statistics.
- Uses evidence of students' current understanding of probability and statistics to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within probability and statistics.
- Designs challenging and engaging problem-solving tasks that develop probability and statistics content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates probability and statistics within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of probability and statistics.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 018

The Master Mathematics Teacher EC–4 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of probability and statistics concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve probability and statistics concepts, skills, and procedures.
- Knows how to teach probability and statistics concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of probability and statistics.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning probability and statistics.
- Evaluates and modifies instruction to improve learning of probability and statistics concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN V—MATHEMATICAL PROCESSES, PERSPECTIVES, MENTORING, AND LEADERSHIP

Competency 019

The Master Mathematics Teacher EC–4 understands and uses mathematical processes to reason mathematically and solve problems.

The Master Mathematics Teacher:

- Demonstrates an understanding of the use of logical reasoning to evaluate mathematical conjectures and justifications and to provide convincing arguments or proofs for mathematical theorems.
- Applies correct mathematical reasoning to derive valid conclusions from a set of premises, and recognizes examples of fallacious reasoning.
- Demonstrates an understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures.
- Applies knowledge of the use of formal and informal reasoning to explore, investigate, and justify mathematical ideas.
- Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
- Evaluates the reasonableness of a solution to a given problem.
- Demonstrates an understanding of estimation and evaluates its appropriate uses.
- Uses physical and numerical models to represent a given problem or mathematical procedure.
- Recognizes that assumptions are made when solving problems; then identifies and evaluates those assumptions.
- Investigates and explores problems that have multiple solutions.
- Applies content knowledge to develop a mathematical model of a real-world situation; then analyzes and evaluates how well the model represents the situation.
- Develops and uses simulations as a tool to model and solve problems.

Competency 020

The Master Mathematics Teacher EC–4 understands mathematical connections, the structure of mathematics, the historical development of mathematics, and how to communicate mathematical ideas and concepts.

The Master Mathematics Teacher:

- Recognizes and uses multiple representations of a mathematical concept.
- Uses mathematics to model and solve problems in other disciplines.
- Uses the structure of mathematical systems and their properties (e.g., mappings, inverse operations) to make connections among mathematical concepts.
- Recognizes the impacts of technological advances on mathematics (e.g., numerical versus analytical solutions) and of mathematics on technology (binary arithmetic).
- Emphasizes the role of mathematics in various careers and professions (e.g., economics, engineering) and how technology (e.g., spreadsheets, statistical software) affects the use of mathematics in various careers.
- Knows and uses the history and evolution of mathematical concepts, procedures, and ideas (e.g., the development of non-Euclidean geometry).
- Recognizes the contributions that different cultures have made to the field of mathematics.
- Uses current and professional resources to plan and develop activities that provide cultural, historical, and technological instruction for the classroom and that connect society and mathematics.
- Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language, and symbolic mathematics.
- Communicates mathematical ideas using a wide range of technological tools and a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
- Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.
- Uses the language of mathematics as a precise means of expressing mathematical ideas.

Competency 021

The Master Mathematics Teacher EC–4 knows how to communicate and collaborate with educational stakeholders to facilitate implementation of appropriate, standards-based mathematics instruction.

The Master Mathematics Teacher:

- Knows the dual role of the Master Mathematics Teacher as teacher and mentor in the school community.
- Knows leadership, communication, and facilitation skills and strategies.
- Knows and applies principles, guidelines, and professional ethical standards regarding collegial and professional collaborations, including issues related to confidentiality.
- Understands the importance of collaborating with administrators, colleagues, parents/guardians, and other members of the school community to establish and implement the roles of the Master Mathematics Teacher and ensure effective ongoing communication.
- Knows strategies for communicating effectively with stakeholders, including other teachers, about using programs and instructional techniques that are based on established research that supports their effectiveness with a range of students, including students who are at-risk.
- Knows strategies for building trust and a spirit of collaboration with other members of the school community to effect positive change in the school mathematics program and mathematics instruction.
- Knows how to use leadership skills to ensure the effectiveness and ongoing improvement of the school mathematics program, encourage support for the program, and engage others in improving the program.
- Knows strategies for collaborating with members of the school community to evaluate, negotiate, and establish priorities regarding the mathematics program and to facilitate mentoring, professional development, and parent/guardian training.
- Knows strategies for conferring with students, colleagues, administrators, and parents/guardians to discuss mathematics-related issues.
- Knows strategies for collaborating with teachers, administrators, and others to identify professional development needs, generate support for professional development programs, and ensure provision of effective professional development opportunities.

Competency 022

The Master Mathematics Teacher EC–4 knows how to provide professional development through mentoring, coaching, and consultation with colleagues to facilitate implementation of appropriate, standards-based mathematics instruction, and makes instructional decisions supported by established research.

The Master Mathematics Teacher:

- Knows and applies skills and strategies for mentoring, coaching, and consultation in the development, implementation, and evaluation of an effective mathematics program.
- Knows learning processes and procedures for facilitating adult learning.
- Knows strategies for facilitating positive change in instructional practices through professional development, mentoring, coaching, and consultation.
- Knows models and features of effective professional development programs that promote sustained applications in classroom practice (e.g., modeling, coaching, follow-up).
- Knows differences between consultation and supervision.
- Knows how to use mentoring, coaching, and consultation to facilitate team building for promoting student development in mathematics.
- Knows how to select and use strategies for collaborating with colleagues to identify needs related to mathematics instruction.
- Knows strategies for collaborating effectively with colleagues with varying levels of skill and experience and/or diverse philosophical approaches to mathematics instruction to develop, implement, and monitor mathematics programs.
- Knows how to select and use strategies to maximize effectiveness as a Master Mathematics Teacher, such as applying principles of time management and engaging in continuous self-assessment.
- Knows sources for locating information about established research on mathematics learning and understands methods and criteria for reviewing research on mathematics learning.
- Knows how to critically examine established research on mathematics learning, analyzes its usefulness for addressing instructional needs, and applies appropriate procedures for translating research on mathematics learning into practice.

SECTION III

APPROACHES TO ANSWERING MULTIPLE-CHOICE ITEMS

The purpose of this section is to describe multiple-choice item formats that you will see on the TExMaT Master Mathematics Teacher (MMT) test and to suggest possible ways to approach thinking about and answering the multiple-choice items. However, these approaches are not intended to replace familiar test-taking strategies with which you are already comfortable and that work for you.

The Master Mathematics Teacher EC–4 test is designed to include 80 scorable multiple-choice items and approximately 10 nonscorable items. Your final scaled score will be based only on scorable items. The nonscorable multiple-choice items are pilot tested by including them in the test in order to collect information about how these questions will perform under actual testing conditions. Nonscorable test items are not considered in calculating your score, and they are not identified on the test.

All multiple-choice questions on this test are designed to assess your knowledge of the content described in the test framework. The multiple-choice questions assess your ability to recall factual information **and** to think critically about the information, analyze it, consider it carefully, compare it with other knowledge you have, or make a judgment about it.

When you are ready to answer a multiple-choice question, you must choose one of four *answer choices* labeled A, B, C, and D. Then you must mark your choice on a separate answer sheet.

In addition to the multiple-choice questions, the MMT test will include one case study assignment. Please see Section V: Case Study Assignment.

Calculators. At the test administration you will be provided with a calculator that can perform the following operations: addition, subtraction, multiplication, division, square root, and percent. The model provided is subject to change. **You may not bring your own calculator to the test.**

Calculators may be used for both the multiple-choice and case study sections of the test.

Definitions and Formulas. A set of definitions and formulas will be provided in your test booklet. A copy of the definitions and formulas is also provided in Section IV of this preparation manual.

Multiple-Choice Item Formats

You may see the following two types of multiple-choice questions on the test.

- Single items
- Items with stimulus material

You may have two or more items related to a single stimulus. This group of items is called a cluster. Following the last item of a clustered item set containing two or more items, you will see the graphic illustrated below.



This graphic is used to separate these clustered items related to specific stimulus material from other items that follow.

On the following pages, you will find descriptions of these commonly used item formats, along with suggested approaches for answering each type of item. In the actual testing situation, you may mark the test items and/or write in the margins of your test booklet, **but your final response must be indicated on the answer sheet provided.**

SINGLE ITEMS

In the single item format, a problem is presented as a direct question or an incomplete statement, and four answer choices appear below the question. The following question is an example of this type. It tests knowledge of Master Mathematics Teacher EC–4 competency 005: *The Master Mathematics Teacher EC–4 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of number concepts, skills, and procedures.*

A class has just finished an introductory lesson on place value, and the teacher wants to assess the effectiveness of the lesson. Which of the following activities would best help the teacher analyze the students' ability to apply the new concept correctly?

- A. Students locate mixed numbers on a number line.
- B. Students find equivalent pairs of fractions and decimals.
- C. Students try to build the greatest number possible with a given set of digits.
- D. Students multiply two- and three-digit numbers by ten and locate them on a number line.

Suggested Approach

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice, and mark it on your answer sheet.

The concept of place value refers to the fact that in a numeration system, the value of a number is determined not only by its digits, but also by the position of the digits in the number. Thus, in a base-ten system, the 2 in 247 actually represents 200 rather than 2, because of its location in the number. Students who understand this will realize that with a given set of digits, the largest number can be written by listing the digits in descending order from left to right. Therefore, the correct response is option C.

The activities described in options A and D analyze students' abilities to order various representations of numbers on a number line, and the activity described in option B analyzes students' abilities to recognize equivalent representations of rational numbers, but none of these activities directly addresses the concept of place value.

ITEMS WITH STIMULUS MATERIAL

Some questions are preceded by stimulus material that relates to the item. Some types of stimulus material included on the test are reading passages, graphics, tables, or a combination of these. In such cases, you will generally be given information followed by an event to analyze, a problem to solve, or a decision to make.

One or more items may be related to a single stimulus. You can use several different approaches to answer these types of questions. Some commonly used approaches are listed below.

Strategy 1 Skim the stimulus material to understand its purpose, its arrangement, and/or its content. Then read the item and refer again to the stimulus material to verify the correct answer.

Strategy 2 Read the item *before* considering the stimulus material. The content of the item will help you identify the purpose of the stimulus material and locate the information you need to answer the question.

Strategy 3 Use a combination of both strategies; apply the "read the stimulus first" strategy with shorter, more familiar stimuli and the "read the item first" strategy with longer, more complex, or less familiar stimuli. You can experiment with the sample items in this manual and then use the strategy with which you are most comfortable when you take the actual test.

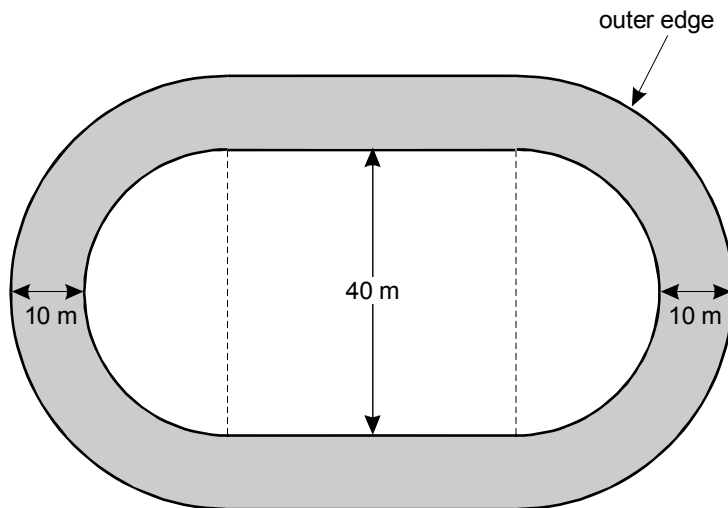
Whether you read the stimulus before or after you read the item, you should read it carefully and critically. You may want to underline its important points to help you answer the item.

As you consider items set in educational contexts, try to use the identified teacher's point of view to answer the items that accompany the stimulus. Be sure to consider the items in terms of only the information provided in the stimulus—not in terms of specific situations or individuals you may have encountered.

Suggested Approach

First read the stimulus (a description and diagram of a running track). A sample stimulus is shown below.

Use the information below to answer the two questions that follow.



A running track, represented in the above diagram by the shaded region, has a width of 10 meters. It surrounds a central area that consists of two semicircular regions attached to a square region.

Now you are prepared to address the first of the two questions associated with this stimulus. The first question measures Master Mathematics Teacher EC–4 competency 0011: *The Master Mathematics Teacher EC–4 understands the basic concepts and applications of Euclidean geometry.*

What is the approximate perimeter of the outer edge of the track, in meters?

- A. 206
 - B. 237
 - C. 268
 - D. 288
-

The perimeter of the outer track consists of two sides of the interior square that has a side length of 40 meters, plus two semicircles of radius 30 meters (half the length of the side of the interior square plus the width of the track). The perimeter of the two semicircles taken together is equivalent to the perimeter of a circle with a radius of 30 meters. The perimeter of a circle can be calculated as $2\pi r$ where r is the radius of the circle, so the total outer perimeter of the track can be represented as $2(40) + 2\pi(30)$, or approximately 268 meters.

Of the four options offered, only option C is correct. Option A results from calculating the outer perimeter using a circle with a radius of 20 meters, and option B from calculating the perimeter using a circle with a radius of 25 meters. Option D results from misinterpreting the diagram and adding 10 meters to the length of the straight section of the track.

Now you are ready to answer the next question. The second question measures competency 0011: *The Master Mathematics Teacher EC–4 understands the basic concepts and applications of Euclidean geometry.*

What is the approximate area of the track (shaded region) in square meters?

- A. 1,571
 - B. 2,371
 - C. 4,884
 - D. 7,083
-

The area of the shaded region can be represented as the sum of the areas of two rectangles, i.e., $2(40 \text{ meters} \times 10 \text{ meters})$, and the area between two concentric circles. The area between the two concentric circles can be calculated as the difference between the area of a circle of radius 30 meters (half the length of the side of the interior square plus the width of the track) and the area of a circle of radius 20 meters (half the length of the side of the interior square). The formula for representing the area of a circle is πr^2 where r is the radius of the circle. Thus the total area of the shaded area in the diagram is calculated as $2(40 \times 10) + \pi(30)^2 - \pi(20)^2$, which equals approximately 2,371 square meters.

Of the four options offered, only option B is correct. Option A calculates the area without including the straight sections of the track. Option C uses the sum of the areas of the two circles instead of the difference between them, and option D uses the diameters instead of the radii of the circles in calculating the areas of the curved sections of the track.

SECTION IV

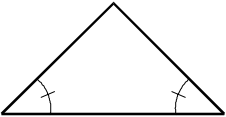
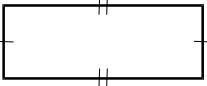
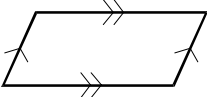
SAMPLE MULTIPLE-CHOICE ITEMS

This section presents some sample multiple-choice items for you to review as part of your preparation for the test. To demonstrate how each competency may be assessed, each sample item is accompanied by the competency number that it measures. While studying, you may wish to read the competency before and after you consider each sample item. Please note that the competency numbers will not appear on the actual test form.

An answer key follows the sample items. The answer key lists the item number and correct answer for each sample item. Please note that the answer key also lists the competency assessed by each item and that the sample items are not necessarily presented in competency order.

The sample items are included to illustrate the formats and types of items you will see on the test; however, your performance on the sample items should not be viewed as a predictor of your performance on the actual examination.

Definitions and Formulas

<p style="text-align: center;">PROBABILITY</p> <p>$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$</p> <p>$P(A \text{ and } B) = P(A) P(B A) = P(B) P(A B)$</p>	<p style="text-align: center;">ALGEBRA</p> <p>Slope $\frac{y_2 - y_1}{x_2 - x_1}$</p> <p>Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</p>
<p style="text-align: center;">GEOMETRY</p> <p>Congruent Angles</p>  <p>Congruent Sides</p>  <p>Parallel Sides</p>  <p>Circumference of a Circle</p> <p style="text-align: center;">$C = 2\pi r$</p>	<p style="text-align: center;">VOLUME</p> <p>Cylinder: (area of base) \times height</p> <p>Cone: $\frac{1}{3}$ (area of base) \times height</p> <p>Sphere: $\frac{4}{3} \pi$ (radius)³</p> <p>Prism: (area of base) \times height</p> <p style="text-align: center;">AREA</p> <p>Triangle: $\frac{1}{2}$ base \times height</p> <p>Rhombus: $\frac{1}{2}$ diagonal₁ \times diagonal₂</p> <p>Trapezoid: $\frac{1}{2}$ height (base₁ + base₂)</p> <p>Sphere: $4\pi r^2$</p> <p>Circle: πr^2</p> <p>Lateral surface area of cylinder: $2\pi rh$</p>

END OF DEFINITIONS AND FORMULAS

MASTER MATHEMATICS TEACHER EC-4

Competency 001

1. Use the two numbers below to answer the question that follows.

$$N = 6 \times 10^2 + 4 \times 10^1 + 7 \times 10^0 + 3 \times 10^{-1}$$

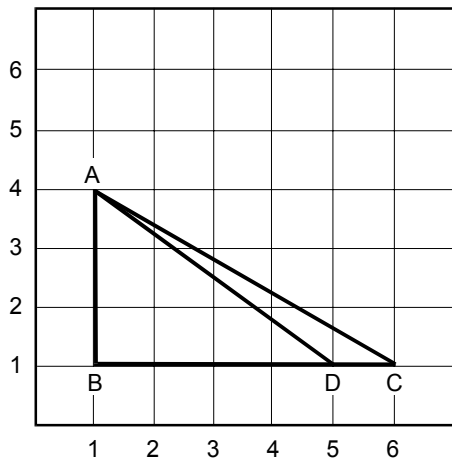
$$M = 1 \times 10^2 + 4 \times 10^0 + 9 \times 10^{-1}$$

The numbers N and M are given above. If the product of N and M is expressed in expanded notation, what is the value of n in the term $n \times 10^2$?

- A. 3
- B. 6
- C. 7
- D. 9

Competency 001

2. Use the diagram below to answer the question that follows.



Which of the following measurements associated with the triangles in the diagram above can be expressed as a nonterminating, nonrepeating decimal?

- A. AD
- B. AC
- C. BC
- D. DC

Competency 001

3. What is the reciprocal of the mixed numeral $5\frac{3}{b}$, where b is a whole number?

- A. $\frac{b}{15}$
- B. $\frac{b}{8}$
- C. $\frac{5b}{5b + 3}$
- D. $\frac{b}{5b + 3}$

Competency 002

4. A student is having difficulty with multiplication problems that involve a two-digit number times a one-digit number. Three examples of the student's work are shown below.

$\begin{array}{r} 2 \\ 27 \\ \times 4 \\ \hline 168 \end{array}$	$\begin{array}{r} 2 \\ 34 \\ \times 6 \\ \hline 304 \end{array}$	$\begin{array}{r} 1 \\ 43 \\ \times 5 \\ \hline 255 \end{array}$
--	--	--

Which of the following problems contains the same error as that made by the student in the three problems above?

A.

$$\begin{array}{r} 4 \\ 26 \\ \times 7 \\ \hline 422 \end{array}$$

B.

$$\begin{array}{r} 3 \\ 35 \\ \times 6 \\ \hline 540 \end{array}$$

C.

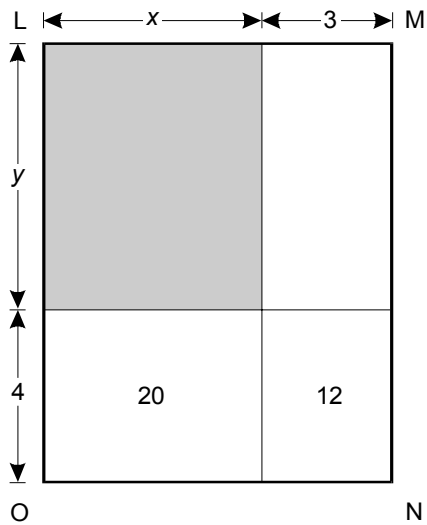
$$\begin{array}{r} 2 \\ 43 \\ \times 8 \\ \hline 324 \end{array}$$

D.

$$\begin{array}{r} 1 \\ 52 \\ \times 7 \\ \hline 704 \end{array}$$

Competency 002

5. Use the diagram below to answer the question that follows.



If the area of rectangle $LMNO$ is 80 , what is the area of the shaded rectangle?

- A. 30
- B. 35
- C. 36
- D. 48

Competency 003

6. A rectangular floor that measures 78 inches by 102 inches is to be completely covered with square tiles. The tiles are available in sizes with only whole-number side lengths. What is the smallest number of uncut tiles that could be used to cover the floor?

- A. 204
- B. 221
- C. 1,326
- D. 3,978

Competency 003

7. If the number 504 is written as a product of its prime factors in the form a^3b^2c , which of the following is the numerical value of $a + b + c$?
- A. 5
 - B. 6
 - C. 12
 - D. 15

Competency 004

8. Students in a fourth-grade class are given the following problem.

A farmer would like to enclose a 24-square-foot rectangular region with fencing. If the length and width of the rectangle are whole numbers, what is the least amount of fencing the farmer could use?

This problem would be most appropriate to use in a unit addressing which of the following?

- A. Find factor pairs and products using arrays and area models.
- B. Make generalizations from patterns and sets of examples.
- C. Use multiplication to solve problems involving two-digit numbers.
- D. Describe shapes and solids in terms of vertices, edges, and faces.

Competency 004

9. Third-grade students are asked to figure out three different ways to calculate the perimeter of an 8×7 rectangle. The students develop the following three methods, and note that they yield the same result.

- add each side ($8 + 7 + 8 + 7$)
- multiply the length and width by two, and add the products $(2 \times 8) + (2 \times 7)$
- add the length and width and multiply the sum by two $2(8 + 7)$

This activity is most likely to promote students' understanding of:

- A. number properties such as the distributive property.
- B. congruence relationships for quadrilaterals.
- C. translating between numeric and symbolic expressions.
- D. prime and composite factors.

Competency 005

10. A student has been encountering some difficulty with subtraction problems involving three-digit numbers. Shown below are four examples of the student's work.

$\begin{array}{r} 6\cancel{7}4 \\ - 259 \\ \hline 415 \end{array}$	$\begin{array}{r} 4\cancel{5}86 \\ - 392 \\ \hline 194 \end{array}$	$\begin{array}{r} 5\cancel{7}45 \\ - 486 \\ \hline 169 \end{array}$	$\begin{array}{r} 2\cancel{4}34 \\ - 276 \\ \hline 68 \end{array}$
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Which of the following would best address the student's errors?

- A. Have the student practice subtraction facts with flashcards.
- B. Use base-ten rods to demonstrate the standard algorithm to the student.
- C. Review subtraction problems involving two-digit numbers with the student.
- D. Use base-ten blocks to demonstrate to the student trading a ten for ones and trading a hundred for tens.

Competency 006

11. Use the table below to answer the question that follows.

Height (cm)	4	10	19
Time (weeks)	1	3	6

A plant grows at a steady rate. The table above gives the height of the plant after several different weeks of growth. What is the height of the plant after 10 weeks?

- A. 28 cm
- B. 30 cm
- C. 31 cm
- D. 33 cm

Competency 006

12. Which of the following situations is best modeled by a relation of the form $y = kx$ where y and x are variables and k is a constant?
- A. John needs to travel 5 miles. He could walk, ride his bike, or take a taxi. How does the length of travel time depend on the speed of travel?
 - B. A store is having a sale in which all items in the store are 80% of the regular price. How does the discounted price of an item depend on the regular price of the item?
 - C. Students are cutting out square pieces of cardboard. How does the area of each square depend on the length of the side of the square?
 - D. A plumbing service charges \$40 per hour and a fixed fee of \$15 for house calls. How does the total cost of a house call depend on the number of hours needed to fix the plumbing problem?

Competency 007

13. Use the calendar below to answer the question that follows.

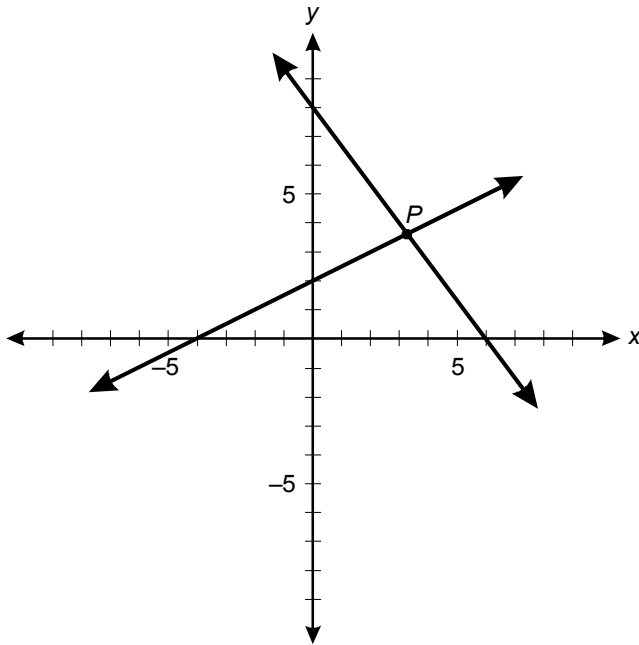
NOVEMBER						
SUN	MON	TUE	WED	THU	FRI	SAT
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

A student notices that in any two-by-two box of four dates on a calendar, the two diagonals of the box sum to the same number. If n represents the largest number in a two-by-two box of four dates, which of the following equations expresses this relationship?

- A. $n + (n - 7) = (n - 1) + (n - 6)$
- B. $n + 2(n - 7) = (n - 1) + 2(n - 6)$
- C. $n + (n - 8) = (n - 1) + (n - 7)$
- D. $n + 2(n - 8) = (n - 1) + 2(n - 7)$

Competency 007

14. Use the graph below to answer the question that follows.



The lines above represent a system of two linear equations. Which of the following equations could be solved to find the x-coordinate of point P ?

- A. $-\frac{8}{6}x + 8 = \frac{1}{2}x + 2$
- B. $-\frac{6}{8}x + 6 = 2x - 4$
- C. $\frac{8}{6}x + 8 = \frac{1}{2}x + 2$
- D. $\frac{6}{8}x + 6 = 2x - 4$

Competency 008

15. Use the learning objective in the box below to answer the question that follows.

The student identifies, extends, and creates patterns.

Which of the following activities best supports this learning objective for kindergarten children?

- A. learning to clap simple rhythms
- B. sorting objects based on shape and size
- C. recognizing symmetry in the letters of the alphabet
- D. identifying basic geometric shapes in the classroom

Competency 008

16. A teacher gives students the sequences below and asks them to find the next term and then explain the rule that determines each next term.

2, 3, 5, 8, 12, 17, 23, . . .

1, 3, 6, 10, 15, 21, 28, . . .

Several students are unable to find the next term or to determine a rule to use. Which of the following suggestions could the teacher make to help these students?

- A. Add the two previous terms.
- B. Divide each term by the previous term.
- C. Look at the difference between successive terms.
- D. Determine the prime factors of each term.

Competency 008

17. A teacher asks students to create a table showing the number of birds in one column and the number of wings in the second column. This exercise is most appropriate for addressing which of the following learning objectives?
- A. The student uses patterns to describe relationships and make predictions.
 - B. The student uses attributes to identify, compare, and contrast shapes and solids.
 - C. The student organizes data to make the data useful for interpreting information.
 - D. The student adds and subtracts whole numbers to solve problems.

Competency 009

18. As a group activity, students are asked to think about how they would write instructions to tell a computer to perform the following tasks.

- Given a person's age, figure out how old the person will be in ten years.
- Given a person's age in years, figure out the person's age in weeks.
- Given two people's ages, figure out the age difference between them.

This activity would most likely benefit students by helping them:

- A. understand how algebraic expressions can be used to represent actual situations and problems.
- B. develop a facility with mental transformation of algebraic expressions.
- C. realize that technology can simplify certain kinds of computational tasks.
- D. recognize equivalent ways of expressing linear functions in symbolic form.

Competency 009

19. Some numeric palindromes are derived in the following manner. Pick an initial number, 27 for example, and add it to its reverse, 72. This produces 99, a numeric palindrome. A teacher wants students to find other numbers such that when each is added to its reverse, it produces the same numeric palindrome. Which of the following questions, posed by the teacher, would be most appropriate to achieve this goal?
- A. What does a palindrome look like if written using expanded notation?
 - B. Is there a relationship between the sum of the initial number's digits and its derived palindrome?
 - C. What is the relationship between the product of a number and its reverse?
 - D. Is there a relationship between a palindrome and its square?

Competency 010

20. Which of the following statements is reasonable?
- A. The volume of a car is about two hundred cubic meters.
 - B. A bicycle weighs about five hundred grams.
 - C. The perimeter of a classroom is about a million centimeters.
 - D. A man has lived for over a billion seconds.

Competency 010

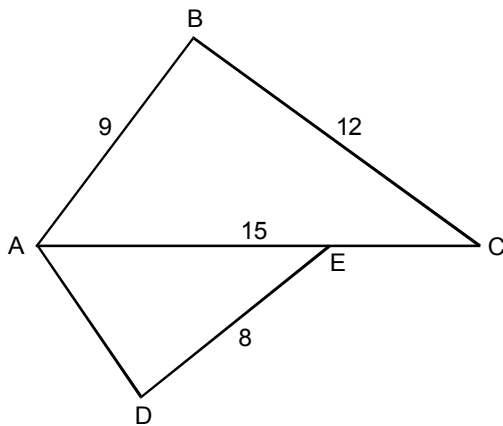
21. The area of a picture frame is 127 square inches. If 1 inch = 2.54 centimeters, what is the area of the frame in square centimeters?
- A. 19.6850
 - B. 50
 - C. 322.58
 - D. 819.3532

Competency 011

22. A hollow right circular cylinder with open ends has a radius of r and a height of $2r$. The cylinder is cut along one side and unrolled to form a rectangle. What are the dimensions of the rectangle?
- A. πr and $2r$
 - B. $2\pi r$ and $2r$
 - C. πr and $2\pi r$
 - D. $2\pi r$ and $4\pi r$

Competency 012

23. Use the diagram below to answer the question that follows.



Triangle ABC is similar to triangle ADE . What is the length of segment EC if $AC = 15$?

- A. 1.7
- B. 5
- C. 6
- D. 11

Competency 012

24. The vertices of a polygon drawn on an x - y coordinate system are given by the ordered pairs below.

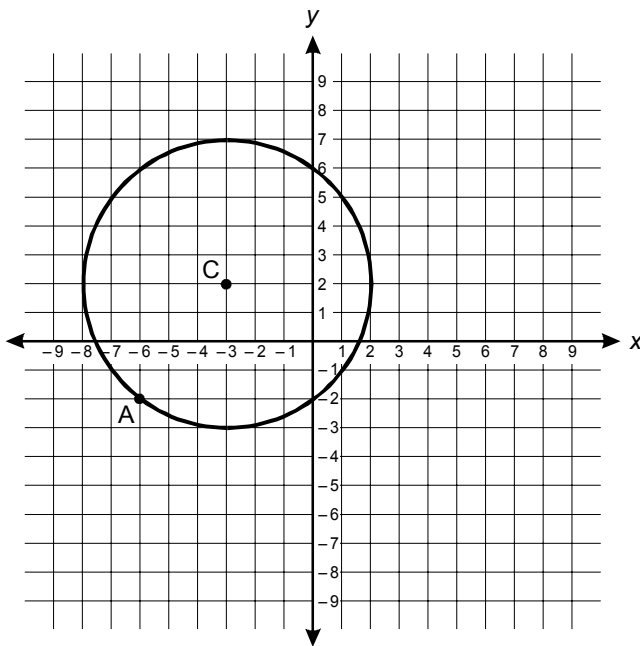
$(1, 2), (2, 4), (5, 2), (4, 4)$

What type of polygon is described by these coordinates?

- A. kite
- B. rectangle
- C. rhombus
- D. trapezoid

Competency 012

25. Use the diagram below to answer the question that follows.



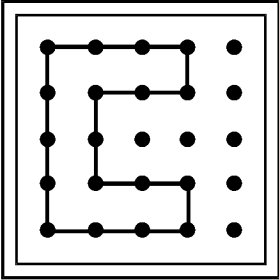
A student places a compass point at C and draws a circle through point A. Which of the following statements best describes this circle?

- A. the set of all points in the plane at a distance of 5 units from $(2, -3)$
- B. the set of all points in the plane at a distance of 5 units from $(-3, 2)$
- C. the set of all points in the plane at a distance of 4 units from $(-3, 2)$
- D. the set of all points in the plane at a distance of 4 units from $(-3, -2)$

Competency 013

26. Use the activity in the box below to answer the question that follows.

On your geoboard, construct 5 figures congruent to the figure shown below.



This activity is most useful for assessing a student's ability to:

- A. calculate areas.
- B. construct similar figures.
- C. identify types of symmetry.
- D. apply translations, reflections, and rotations.

Competency 013

27. A teacher asks students to separate cardboard models of several different polygons into two groups depending on whether the polygons are concave or convex. Several students are having difficulty distinguishing the difference between these types of polygons. Which of the following activities would most likely help the students understand the difference between these types of polygons?

- A. Have students measure the interior angles of the polygons to determine if they are equal.
- B. Have the students wrap a string around the polygons and describe their observations.
- C. Have the students trace the polygons on grid paper and count the number of squares inside each polygon.
- D. Have the students try to fold the polygons in half so that the two halves match each other without overlapping.

Competency 014

28. A teacher gives a student the following problem.

One can of Chewy Nuts holds 2 pounds 5 ounces of nuts. A case of Chewy Nuts contains a dozen cans. How many pounds of nuts are in a case of Chewy Nuts?

The student answers that a case contains 30 pounds of nuts. Which of the following questions should the teacher ask first in order to help the student understand his or her mistake?

- A. What is 5 times 12?
- B. How many units are in a dozen?
- C. What is the remainder of $60 \div 16$?
- D. How many ounces are in a pound?

Competency 014

29. Students use the lengths of their hands as units to measure common objects in the classroom. Each student then enters the number of units for each of the various items measured in a chart on the chalkboard. The teacher would most likely be using this activity to emphasize that, for a measurement unit to be functional, the:

- A. units used must be identical.
- B. measuring tool must be one unit long.
- C. units must be smaller than the item measured.
- D. measuring tool must be convenient to use.

Competency 015

30. Use the problem in the box below to answer the question that follows.

Six people have been nominated to serve on a two-person committee. How many different two-person committees could be selected from the group of six nominees?

Which of the following methods for solving this problem will give a correct answer?

- A. Recognize that each person could be paired with each of the other people. Therefore, there are six times five, or thirty, different two-person committees.
- B. Solve a simpler version of the problem for a group of three nominees. Multiply the answer to this simpler problem by two.
- C. Draw six dots in a circular pattern. Represent each two-person committee by drawing a line connecting each pair of the dots; then count the lines.
- D. Use six different-colored chips to represent the six people. Separate the chips into groups of two; then count the number of groups that result.

Competency 015

31. A student is playing a game with a set of cards on which are printed the 21 consonants and 5 vowels that compose the English alphabet. The student turns the cards face down, shuffles them, and draws a card from the top of the stack. Given that the student has drawn a vowel, what is the probability that the letter is *A* or *E*?

- A. $\frac{1}{26}$
- B. $\frac{1}{13}$
- C. $\frac{1}{5}$
- D. $\frac{2}{5}$

Competency 015

32. Leah is going on a four-day vacation to a city that has a 30% chance of rain each day. To determine the probability that it will rain on two or more of the four days of her vacation, she creates the following simulation.

Write the numbers 1 through 10 on slips of paper and put the slips in a box. Let the numbers 1, 2, and 3 represent rainy days. Without looking, draw a slip from the box, record the number, and throw the slip away. Repeat this procedure four times. If two or more of the four slips had the number 1, 2, or 3, count the trial as a positive result. Repeat this trial many times, and calculate the fraction of the trials that were positive.

For this to be an accurate simulation, it should be changed in which of the following ways?

- A. After drawing a slip, replace the slip in the box.
- B. Count the trial as a positive result if any of the four slips had the number 1, 2, or 3.
- C. Do only a single trial, rather than many trials.
- D. Count the trial as a positive result only if exactly two of the four slips had the number 1, 2, or 3.

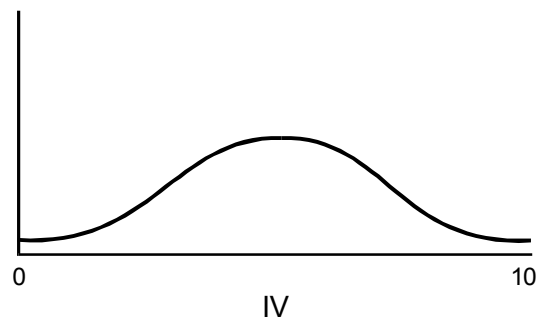
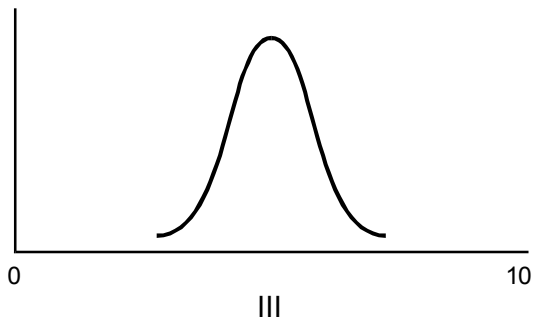
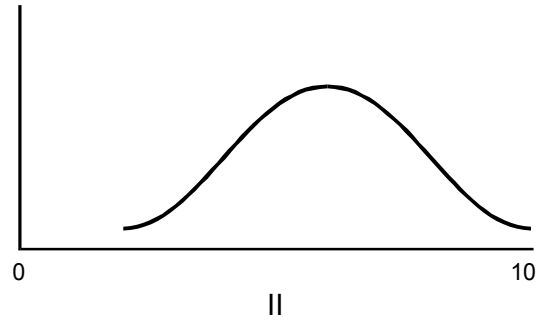
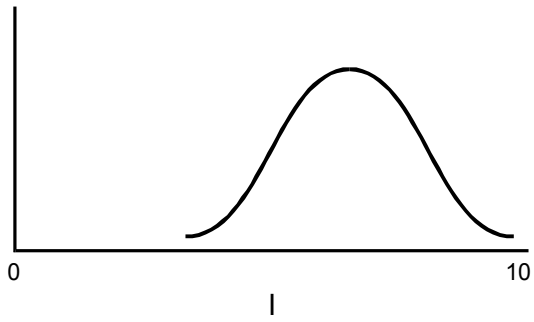
Competency 016

33. An elementary school is divided into three buildings: Building A, Building B, and Building C. In the school, 40% of the students are in Building A and 32% of the students are in Building B. When these data are presented in a circle graph, what is the measure of the central angle representing the number of students in Building C?

- A. 28°
- B. 72°
- C. 100.8°
- D. 129.6°

Competency 016

34. Use the graphs of distributions below to answer the question that follows.



Which of the following distributions corresponds to the data set with the largest standard deviation?

- A. I
- B. II
- C. III
- D. IV

Competency 017

35. Students in a fourth-grade class are engaged in a partner activity in which they draw a marble from a bag with five red marbles and five blue marbles, replace the marble, shake the bag, and draw again. The teacher overhears a student telling his partner that since they've drawn six red marbles in a row, they're likely to get a blue marble on the next draw. This student would most benefit from instruction designed to promote understanding of which of the following concepts?
- A. addition and multiplication properties of probability
 - B. sample space
 - C. dependent and independent events
 - D. complementary events

Competency 017

36. **Use the information below to answer the question that follows.**

At the beginning of the year, a kindergarten teacher has the students mark their birthdays by placing photographs of themselves on an oversized calendar at the front of room. Later, the class figures out which month has the most birthdays and which has the fewest.

This activity is aligned with which of the following learning objectives?

- A. The student constructs and uses graphs and other representations of real objects to answer questions.
- B. The student describes the order of events or objects.
- C. The student uses logical reasoning to make sense of his or her world.
- D. The student identifies, extends, and creates patterns.

Competency 018

37. Which of the following is an appropriate strategy for introducing first-grade students to the concepts involved in probability?
- A. Give students data on the class's favorite desserts and ask them to make a bar graph that shows the likelihood that ice cream is a student's favorite dessert.
 - B. Use the weather as an example of how probability is used in real life to predict what is likely to happen in the future.
 - C. Have the students play a dice game to create real data and then explain the likelihood of getting a particular score.
 - D. Create a likelihood line that covers impossible, unlikely, likely, and certain events and ask students to describe an event for each category.

Competency 018

38. A student determines that there is a one-in-four chance that three consecutive coin flips will all come up the same (i.e., all heads or all tails). The student then conducts eight separate experimental trials of three coin flips each. However, none of the eight trials results in all heads or all tails. To help the student reconcile her calculation with the experimental results, the teacher should instruct the student to:
- A. perform a much larger number of experimental trials.
 - B. consider whether she should have added probabilities, rather than multiplying.
 - C. use a tree diagram to check her calculation.
 - D. try a different coin in case the coin she is using is biased.

Competency 019

39. Which of the following is an example of inductive reasoning?
- A. Before determining the average of a set of data, a student recalls the distinction between mean, median, and mode.
 - B. After learning that a square is a rectangle and a rectangle is a parallelogram, a student concludes that a square is a parallelogram.
 - C. To determine whether 37 is a prime number, a student lists each of the smaller whole numbers and considers whether any of them are factors of 37.
 - D. After multiplying several pairs of whole numbers, a student forms the hypothesis that only two odd numbers can have a product that is odd.

Competency 019

40. **Use the information below about the members of a set to answer the question that follows.**

- No odd numbers are in the set.
- All members of the set are greater than 34.
- If a number is in the set, it is a multiple of 5.

Which of the following is a valid conclusion from the above premises?

- A. The set is empty.
- B. All members of the set are greater than 39.
- C. The set does not include 90.
- D. All multiples of 5 that are greater than 34 are in the set.

Competency 019

41. Use the information below to answer the question that follows.

Choose a number.	<input type="checkbox"/>
Add five.	<input type="checkbox"/> ○ ○ ○ ○ ○
Double the result.	<input type="checkbox"/> ○ ○ ○ ○ ○ <input type="checkbox"/> ○ ○ ○ ○ ○
Subtract four.	<input type="checkbox"/> ○ ○ ○ ○ <input type="checkbox"/> ○ ○ ○ ○
Divide by two.	<input type="checkbox"/> ○ ○ ○
Subtract the original number.	○ ○ ○
The result is three.	

The information in the box above shows a student's explanation for why a given number trick works. In providing this explanation, the student has demonstrated an ability to:

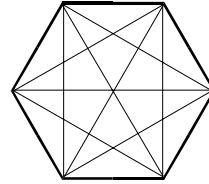
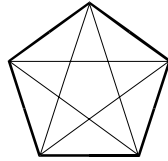
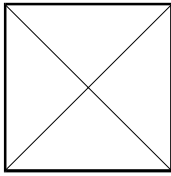
- A. prove a statement using deductive reasoning.
- B. develop a conjecture using inductive reasoning.
- C. solve a problem using proportional reasoning.
- D. validate a statement using indirect reasoning.

Competency 020

42. An economist is studying the relationship between the amount workers in a factory are paid per hour and the number of hours they choose to work per week. The economist's hypothesis is that as workers' hourly pay increases, they will increase the number of hours of work at a steady rate. For example, if a worker who works 30 hours per week has her pay raised from \$9 to \$10 per hour, she will work 32 hours per week; if her pay is then raised to \$11 per hour, she will work 34 hours per week. In mathematical language, the economist's hypothesis is that the number of hours worked per week is:
- A. a linear function of the worker's hourly wage.
 - B. a constant multiple of the worker's hourly wage.
 - C. a quadratic function of the worker's hourly wage.
 - D. inversely proportional to the worker's hourly wage.

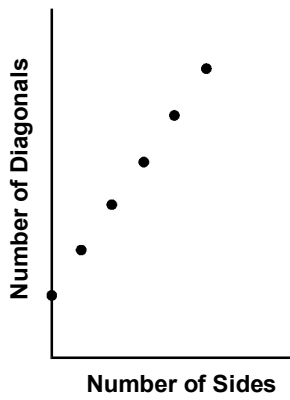
Competency 020

43. Use the diagrams below to answer the question that follows.

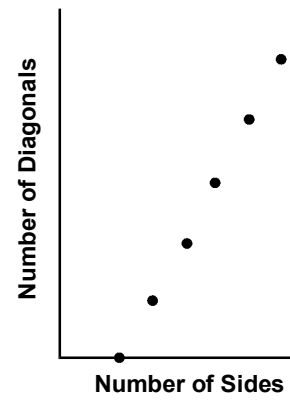


The diagrams above show several regular polygons with all possible diagonals drawn. Which of the following graphs best represents the relationship of the number of sides of a polygon to the number of diagonals?

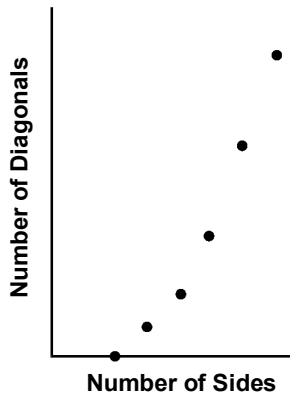
A.



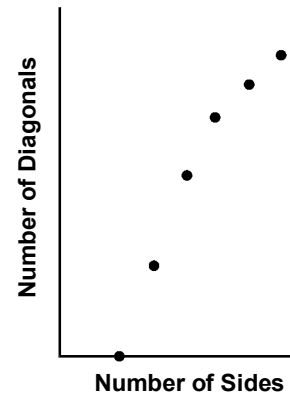
B.



C.



D.



Competency 021

44. In addition to his or her own classroom teaching duties, a Master Mathematics Teacher could reasonably expect to become involved in:
- A. performing regular evaluations of other mathematics teachers in the school.
 - B. regularly co-teaching with a classroom teacher who is experiencing difficulty implementing the mathematics curriculum.
 - C. diagnosing and working with small groups of students with disabilities.
 - D. collaborating with classroom teachers individually and collectively in improving the quality of mathematics instruction in the school.

Competency 021

45. Members of a team developing an explicit, systematic mathematics program for at-risk students have very different points of view. After the first meeting, several team members express to the Master Mathematics Teacher their concerns about the team's ability to make meaningful changes due to all the differing opinions. Which of the following would be the most appropriate action for the Master Mathematics Teacher to take at this point?
- A. Acknowledge the diverse views and point out that differences of opinion can strengthen the work of a team by presenting members with a variety of solutions to consider.
 - B. Suggest that the members help organize a special meeting of the team to discuss the different opinions and establish guidelines for achieving compromise.
 - C. Speak individually with each team member about the differences in points of view and encourage any who feel they might be unable to accept compromises to withdraw.
 - D. Request a meeting with the school principal to explain the conflicting views and suggest that the principal take a stronger stand in support of the new program.

Competency 022

46. An experienced second-grade teacher has started using manipulatives to teach number concepts. She asks the Master Mathematics Teacher to help assess the effectiveness of the new teaching method by observing her class as a peer mentor. The Master Mathematics Teacher, after attending the class, wants to provide helpful feedback to the second-grade teacher. Which of the following would be the most appropriate way in which this can be accomplished?
- A. Encourage discussion of why some aspects of the lesson were successful and others were not, using examples and anecdotes to describe what the teacher did and how the students responded.
 - B. Ask the teacher to describe in her own words how she felt about the lesson and then provide her with a list of resources on using manipulatives in the classroom.
 - C. Begin discussion by mentioning the more successful elements of the lesson and then point out some of the problems along with possible strategies for their improvement.
 - D. Emphasize that the lesson had many positive aspects and then show the teacher several specific ways to use manipulatives to meet her learning goals.

Competency 022

47. The Master Mathematics Teacher is observing a new mathematics teacher, Mr. Sanborn. Although the students appear to understand the concepts being taught, the Master Mathematics Teacher is concerned that they may lack the basic skills necessary to perform according to district curriculum standards. Which of the following would be the most appropriate way for the Master Mathematics Teacher to address his concerns?
- A. Acknowledge Mr. Sanborn's effective teaching strategy and also encourage him to assess his students' learning in terms of curriculum-based computation standards.
 - B. Point out that Mr. Sanborn's lessons do not specifically address curriculum standards and explain that he will need to revise lessons to spend more time teaching basic computation skills.
 - C. Discuss the omission with Mr. Sanborn and offer to provide him with lesson plans to acquaint him with what he needs to fill in the gap in his instruction.
 - D. Bring Mr. Sanborn's style to the attention of the head of the mathematics department and suggest that workshops on teaching computation skills be made a priority for professional development.

ANSWER KEY

Item Number	Correct Answer	Competency
1	D	001
2	B	001
3	D	001
4	A	002
5	A	002
6	B	003
7	C	003
8	A	004
9	A	004
10	D	005
11	C	006
12	B	006
13	C	007
14	A	007
15	A	008
16	C	008
17	A	008
18	A	009
19	B	009
20	D	010
21	D	010
22	B	011
23	B	012
24	D	012

Item Number	Correct Answer	Competency
25	B	012
26	D	013
27	B	013
28	D	014
29	A	014
30	C	015
31	D	015
32	A	015
33	C	016
34	D	016
35	C	017
36	A	017
37	D	018
38	A	018
39	D	019
40	B	019
41	A	019
42	A	020
43	C	020
44	D	021
45	A	021
46	A	022
47	A	022

SECTION V

CASE STUDY ASSIGNMENT

In addition to the multiple-choice section, the Master Mathematics Teacher (MMT) test will include one case study assignment that requires a written response. The written-response score will be combined with the multiple-choice score to produce a total test scaled score.

Included in this section is a description of the case study assignment, an explanation of the way case study assignment responses will be scored, and one sample case study assignment.

How Case Study Assignment Responses Are Scored

Responses will be scored on a four-point scale (see next page). Each point on the scale represents the degree to which the performance characteristics (see below) are demonstrated in the response.

The score point descriptions reflect typical responses at each score point. Although the score assigned corresponds to one of the score points, individual responses may include attributes of more than one score point.

PERFORMANCE CHARACTERISTICS

PURPOSE	The extent to which the candidate responds to the components of the assignment in relation to relevant competencies in the Master Mathematics Teacher EC–4 test framework.
APPLICATION OF KNOWLEDGE	Accuracy and effectiveness in the application of knowledge as described in relevant competencies in the Master Mathematics Teacher EC–4 test framework.
SUPPORT	Quality and relevance of supporting details in relation to relevant competencies in the Master Mathematics Teacher EC–4 test framework.
RATIONALE	Soundness of reasoning and depth of understanding of the assigned task in relation to relevant competencies in the Master Mathematics Teacher EC–4 test framework.
SYNTHESIS	The extent to which the candidate is able to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher EC–4 in an applied context.

SCORE SCALE

Score	Score Point Description
4	<p>The "4" response reflects thorough knowledge and understanding of relevant competencies in the Master Mathematics Teacher EC–4 test framework.</p> <ul style="list-style-type: none"> • The response addresses all components of the assignment and fully completes the assigned task. • The response demonstrates an accurate and very effective application of relevant knowledge. • The response provides strong supporting evidence with specific and relevant examples. • The response demonstrates clear, logical reasoning and a comprehensive understanding of the assigned task. • The response demonstrates strong ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher EC–4.
3	<p>The "3" response reflects sufficient knowledge and understanding of relevant competencies in the Master Mathematics Teacher EC–4 test framework.</p> <ul style="list-style-type: none"> • The response addresses most or all components of the assignment and sufficiently completes the assigned task. • The response demonstrates a generally accurate and effective application of relevant knowledge; minor problems in accuracy or effectiveness may be evident. • The response provides sufficient supporting evidence with mostly specific and relevant examples. • The response demonstrates sufficient reasoning and an overall understanding of the assigned task. • The response demonstrates sufficient ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher EC–4.
2	<p>The "2" response reflects partial knowledge and understanding of relevant competencies in the Master Mathematics Teacher EC–4 test framework.</p> <ul style="list-style-type: none"> • The response addresses at least some components of the assignment and/or partially completes the assigned task. • The response demonstrates a partial and/or ineffective application of relevant knowledge; significant inaccuracies may be evident. • The response provides minimal supporting evidence with few relevant examples; some extraneous or unrelated information may be evident. • The response demonstrates limited reasoning and understanding of the assigned task. • The response demonstrates partial ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher EC–4.
1	<p>The "1" response reflects little or no knowledge or understanding of relevant competencies in the Master Mathematics Teacher EC–4 test framework.</p> <ul style="list-style-type: none"> • The response addresses few components of the assignment and/or fails to complete the assigned task. • The response demonstrates a largely inaccurate and/or ineffective application of relevant knowledge. • The response provides little or no supporting evidence, few or no relevant examples, or many examples of extraneous or unrelated information. • The response demonstrates little or no reasoning or understanding of the assigned task. • The response demonstrates little or no ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher EC–4.
U	<p>The "U" (Unscorable) will be assigned to responses that are off topic/off task, illegible, primarily in a language other than English, or are too short or do not contain a sufficient amount of original work to score.</p>
B	<p>The "B" (Blank) will be assigned to written response booklets that are completely blank.</p>

Note: Your written response should be your original work, written in your own words, and not copied or paraphrased from some other work.

Scoring Process

Case study assignment responses are scored on a scale of 1 to 4. Each response is evaluated by a minimum of two scorers with expertise in mathematics instruction. All scorers have successfully completed standardized orientation and are calibrated to the scoring criteria throughout the scoring session.

Analytic Notation

Examinees who do not pass the test and do not perform satisfactorily on the case study assignment will receive information concerning specific aspects of the written response that show a need for improvement. This information will be provided for examinees to use in preparing to retake the test.

If you do not pass the test or perform satisfactorily on the case study assignment, your score report will indicate one or more of the following areas for improvement in your written response. These areas are based on the performance characteristics in the score scale.

- Purpose
- Application of Knowledge
- Support
- Rationale
- Synthesis

Preparing for the Case Study Assignment

Following is one sample case study assignment that represents the type of question you will see on the MMT test.

In preparing for the case study assignment component of the test, you may wish to draft a response to the question by reading the case study and planning, writing, and revising your essay. You should plan to use about 90 minutes to respond to the sample case study assignment. Also, since no reference materials will be available during the test, it is recommended that you refrain from using a dictionary, a thesaurus, or textbooks while writing your practice response.

After you have written your practice response, review your response in light of the score point descriptions. You may also wish to review your response and the score scale with staff in your MMT preparation program.

General Directions for Responding to the Case Study Assignment

DIRECTIONS FOR CASE STUDY ASSIGNMENT Master Mathematics Teacher EC–4

General Directions:

This section of the test consists of one case study assignment. For this assignment, you are to prepare a written response and record it in the area provided in the written response booklet.

Read the case study assignment carefully before you begin to write. Think about how you will organize what you plan to write. You may use any blank space provided in this test booklet to make notes, create an outline, or otherwise prepare your response. ***Your final response, however, must be written in the written response booklet.***

Evaluation Criteria:

Your written response will be evaluated based on the extent to which it demonstrates the knowledge and skills required to perform the roles of the Master Mathematics Teacher EC–4. You may draw from research and your professional experience. (Citing specific research is not required.)

Read the assignment carefully to ensure that you address all components. Your response to the assignment will be evaluated based on the following criteria:

- **PURPOSE:** The extent to which you respond to the components of the assignment in relation to relevant competencies in the Master Mathematics Teacher EC–4 test framework.
- **APPLICATION OF KNOWLEDGE:** Accuracy and effectiveness in the application of knowledge as described in relevant competencies in the Master Mathematics Teacher EC–4 test framework.
- **SUPPORT:** Quality and relevance of supporting details in relation to relevant competencies in the Master Mathematics Teacher EC–4 test framework.
- **RATIONALE:** Soundness of reasoning and depth of understanding of the assigned task in relation to relevant competencies in the Master Mathematics Teacher EC–4 test framework.
- **SYNTHESIS:** The extent to which you are able to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher EC–4 in an applied context.

The assignment is intended to assess knowledge and skills required to perform the roles of the Master Mathematics Teacher EC–4, not writing ability. Your response, however, must be communicated clearly enough to permit a valid judgment about your knowledge and skills. Your response should be written for an audience of educators knowledgeable about the roles of the Master Mathematics Teacher EC–4.

The final version of your response should conform to the conventions of edited American English. Your response should be your original work, written in your own words, and not copied or paraphrased from some other work. You may, however, use citations when appropriate.

Sample Case Study Assignment

48. **Classroom Context:** This case study focuses on a third-grade teacher, Ms. Kendall, who is teaching her class how to use standard units of measure to find the perimeter of shapes. The mathematics instruction time is 50 minutes a day. The class is composed of students who achieve at various levels.

Master Mathematics Teacher Task: Ms. Kendall has asked the Master Mathematics Teacher (MMT) to observe her class and provide assistance on teaching students how to use measurement to find the perimeter of shapes. The MMT has agreed to observe her lesson. Ms. Kendall shows the MMT a lesson plan that she intends to use on the day of the MMT's observation. On the following pages, you will find:

- background information regarding Ms. Kendall's previous instruction for this class;
- the lesson plan implemented on the day of the MMT's observation;
- an assignment given by Ms. Kendall to her class;
- excerpts of notes taken by the MMT while observing Ms. Kendall's lesson; and
- representative samples of student work from the class.

Using these materials, write a response in which you apply your knowledge of mathematics, mathematics instruction, and mentoring to analyze this case study. Your response should include the following information:

- An analysis of two significant weaknesses in the effectiveness of the lesson on using measurement to find the perimeter of shapes. Cite evidence from the case study to support your observations.
- A full description of two instructional strategies or assignments that would be effective for Ms. Kendall to use to address the weaknesses you have identified. Be sure to describe one strategy or assignment for each of the weaknesses you identified.
- An explanation of why each of the strategies or assignments you have described would be effective in improving Ms. Kendall's instruction on using measurement to find the perimeter of shapes.
- A full description of two appropriate actions you would take as a mentor teacher to help Ms. Kendall implement the strategies or activities you have described.

Information from the teacher regarding previous instruction: Students are in the last quarter of their third-grade year. They have been taught addition, subtraction, and multiplication facts to ten. Students have just recently been introduced to and practiced measuring with inches, feet, and yards. Students have also had experience with naming different shapes.

LESSON PLAN

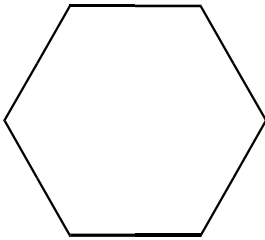
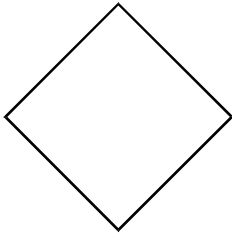
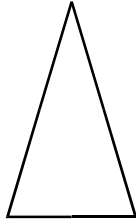
Objective: Students will use standard linear units of measure to find the perimeter of a given shape and make comparisons with perimeters of other shapes.

Warm-up	<ul style="list-style-type: none">• Students are given a pre-cut rectangle and asked to measure each side of the rectangle with their inch rulers and then write the measurements inside the rectangle.
Procedures	<ol style="list-style-type: none">1. Define new math term: perimeter = sum of the lengths of all the sides.2. Discuss reasons people might need/want to know perimeter. (For example, fencing a garden or railings around a parking lot, borders, corral for horses, border on the bulletin board, etc.)3. Show some of these situations in pictures.4. Use a rectangular drawing on the overhead and a transparent ruler to model how to add the side measurements to find the perimeter.5. Students add up the sides of the shape from the warm-up and write the perimeter inside the shape.6. Review what perimeter means and show the correct perimeter and have students check the one that they did.
Classwork	<ol style="list-style-type: none">1. Give groups a pre-cut shape of either a triangle or one of two different rectangles. Groups find perimeter.2. Check work on the board. Ask a student to show how to find the perimeter of a triangle. Then do other shapes.3. Give the "Finding the Perimeter of Shapes" worksheet, which shows representations of the second set of pre-cut shapes (triangle, square, hexagon, rectangle).* Have students write the measurements of the pre-cut shapes on the worksheet.
Homework	<ul style="list-style-type: none">• No homework
Materials	<ul style="list-style-type: none">• Inch ruler, pre-cut shapes, easel

*A copy of the assignment follows this lesson plan.

CLASSWORK

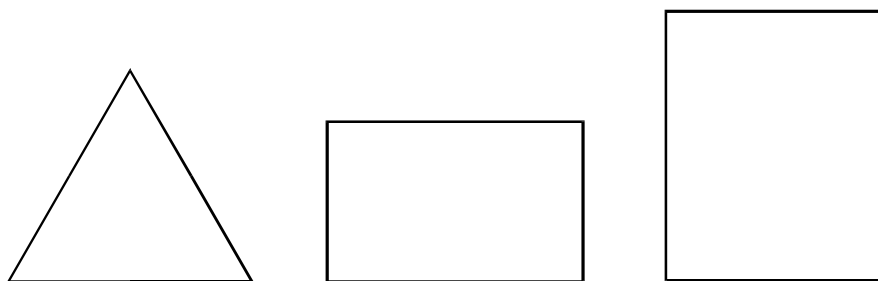
Finding the Perimeters of Shapes*



*Shapes are representations of the pre-cut shapes given to students. Shapes shown above are not drawn to scale.

SELECTED EXCERPTS FROM THE MMT'S OBSERVATION NOTES

- As a warm-up Ms. Kendall asks students to use their inch rulers to find the length of each side of the rectangle the students are given. She circulates around the room as the students are working. Many students appear to be measuring correctly, but a few students are asking for assistance.
- Ms. K calls attention to a chart on which a definition of perimeter is written. She reads the definition and explains that they are going to figure out the perimeter of shapes today.
- Ms. K asks the class to watch the overhead as she measures the sides of a square. She uses a ruler and tells students what the measurements are on each side. She tells them "You must add all four sides to get the perimeter of the shape."
- Students say they cannot see the numbers, and some are becoming inattentive.
- Ms. K holds up three pre-cut shapes--one triangle and two different rectangles (see below). She tells students that each group will be finding the perimeter for one of the shapes.



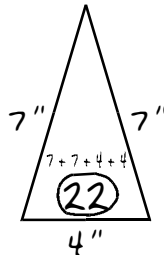
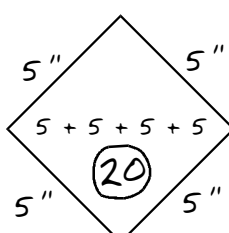
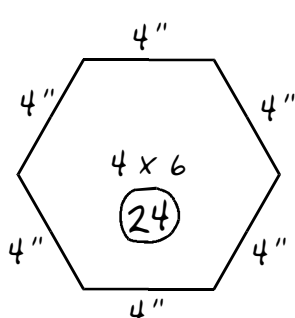
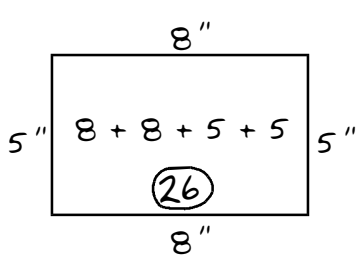

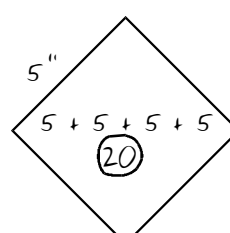
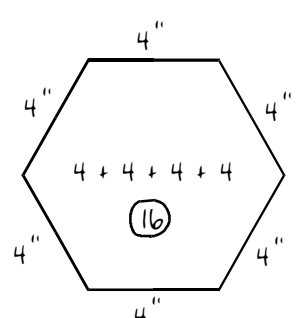
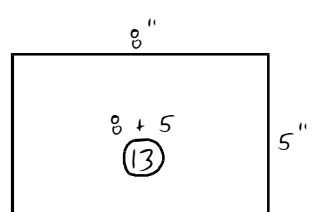
- Each group receives either a triangular shape or one of the two rectangular shapes. The shapes all have the same perimeter.
- Ms. K circulates around the room. She defines perimeter again saying that they must add all four sides to get the perimeter. One student calls out that the shape has three sides. She replies, "Yes, for triangles you add three sides." She assists several groups of students in finding the perimeter of their shape.

SELECTED EXCERPTS FROM THE MMT'S OBSERVATION NOTES (continued)

- Ms. K asks students to gather around an easel. She uses the three examples of student work on the original 3 pre-cut shapes. She tapes these shapes to the paper on the easel and initiates a discussion about perimeter.
- Ms. K asks what each shape has in common. A student says, "Straight sides." She replies, "Anything else?" Another student says, "The perimeters are the same." Ms. K asks what is different about the shapes. "They're not all four-sided figures. They are different sizes." She responds, "So, can we say the perimeter of the shape can be the same even if the size is different?"
- Ms. K: "I wonder if you were building a corral for your horse, would you need more fencing for a corral of shape 1, 2, or 3? Think about that." A few hands are raised, but many students are puzzled. Ms. K says, "Well, they would all be the same because the perimeters are the same for all of the shapes. See . . ." and points to shapes.
- Ms. K says, "Which shape would you use for your corral if your horse loves to run and eat grass?" Several students say that it doesn't matter since they all have the same perimeter. Ms. K replies that they will do some cutting of shapes tomorrow to see if the class can answer the question.
- Ms. K hands out a second set of pre-cut shapes along with the "Finding the Perimeters of Shapes" worksheet. Students are asked to measure the sides of each shape, transfer the measurements to the worksheet, and find the perimeter of each shape. She circulates to observe work and notices there are several students who are struggling.

SAMPLE STUDENT WORK FROM CLASSWORK

*The problems below are representative samples of student work from the class.**

<p>S.J.</p>    	<p>N.M.</p>    
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*Shapes are not drawn to scale.

SECTION VI

PREPARATION RESOURCES

The resources listed below may help you prepare for the TExMaT test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

Journals

Arithmetic Teacher, National Council of Teachers of Mathematics.

Creative Classroom, Children's Television Workshop.

The Elementary School Journal, University of Chicago Press.

Exceptional Children, Council for Exceptional Children.

Instructor, Scholastic, Inc.

Journal for Research in Mathematics Education, National Council of Teachers of Mathematics.

Journal of Computing in Childhood Education, Journal of the Association for the Advancement of Computing in Education.

Teaching Children Mathematics, National Council of Teachers of Mathematics.

Teaching PreK–8, Early Years, Inc.

Young Children, Journal of the National Association for Young Children.

Other Sources

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Online Resources

Mathematics TEKS Toolkit, <http://www.tenet.edu/teks/math>

National Council of Teachers of Mathematics, <http://www.nctm.org>

Texas Education Agency—Math Initiative, <http://www.tea.state.tx.us/math>

